Substructure Isolation and Identification using FFT of Measured Local Responses

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ABSTRACT

A substructuring method is presented for substructure identification and local health monitoring. The concerned substructure is numerically separated from the global structure to be a so-called *Isolated Substructure* by adding virtual supports on the substructure interface. The isolated substructure is a small and independent structure; its virtual supports are constructed using the FFT of measured local responses of the global structure. The damage of the substructure can be then identified easily by any of the classical methods which perform well on global structures. An experiment of a cantilever beam, of which the upper part is chosen as the substructure, is used to validate the method.

INTRODUCTION

In recent years, Structural Health Monitoring (SHM) has become a widely researched field in civil engineering [1]. Recently, global damage identification is becoming increasingly difficult as the structures are becoming larger and more complex. In fact, in many practical applications only local substructures are crucial; in these cases, a substructure-only monitoring would be sufficient as well as advantageous with regard to easier implementation and less cost.

The current substructuring methods focus primarily on building the equation of motion of the considered substructure. However, the core idea of the method described here is different [2-5]. The proposed method consists of two steps: isolation and identification. First, the isolation process is performed here by constructing the frequency responses of the isolated substructure using the Fast Fourier Transform (FFT) of measured local responses of the global structure to an

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impact hammer. Second, local damage is identified by classical frequency-based methods using the eigenfrequencies of the substructure, which are found via the peaks of its constructed frequency responses.

A numerical example, a shear frame with 3 floors, is introduced to describe the isolation process. Then, an experiment of a cantilever beam, of which the upper part is chosen as the substructure, is used to validate the method. Both the isolation and the identification steps are performed very well using the measured responses.

SUBSTRUCTURE ISOLATION METHOD

In agreement with the general approach of the virtual distortion method (VDM) [6,7], the response (in time domain or frequency domain) of an isolated substructure is expressed as a sum of the responses of the global structure to the same load (which is applied inside of the substructure) and to certain virtual loads that are applied on the substructure interface or outside of it in order to model the supports. The virtual loads can be computed using the condition that the considered substructure is really isolated by the virtual supports, so that the interface responses vanish. Thereupon, the corresponding responses of any sensor placed inside the substructure can be constructed by using their measured responses and the responses to the computed virtual loads. The following sections introduce the theory of the isolation using the FFT of the responses in frequency domain.

Isolation Using Frequency Response

There are two kinds of sensors placed on the substructure. If there are n_b degrees of freedom (Dofs) on the substructure interface, n_b sensors should be placed in these Dofs for isolation. Furthermore, n_s sensors are placed inside the substructure to obtain the basic information about the isolated substructure. Let the global structure be externally excited by loads $f_i(\omega)$ (j=1,2,…,n_b.) in frequency domain which can be applied on the substructure interface or outside of it. Denote by $b_{ij}^M(\omega)$ and $d_{\alpha j}^M(\omega)$ the resulting frequency responses of the interface and inner sensors respectively. They are called the isolation responses and shown in the left item of Eq.(1), where $B_{ij}^0(\omega)$ and $D_{\alpha j}^0(\omega)$ is the corresponding frequency response of the respective interface or inner sensor by a unit harmonic load applied on the location of the j-th load.

$$\begin{cases} b_{ij}^M(\omega) = B_{ij}^0(\omega) f_j(\omega) \\ d_{\alpha j}^M(\omega) = D_{\alpha j}^0(\omega) f_j(\omega) \end{cases} \qquad \begin{cases} a_i^M(\omega) = B_{ip}^0(\omega) p(\omega) \\ u_\alpha^M(\omega) = D_{\alpha p}^0(\omega) p(\omega) \end{cases} \tag{1}$$

Apply an external excitation $p(\omega)$ inside the substructure and denote respectively by $a_i^M(\omega)$ and $u_\alpha^M(\omega)$ the resulting frequency responses of the interface and inner sensors, which are called the basic response and shown in the right item of Eq.(1). The corresponding frequency response functions are $B_{ip}^0(\omega)$ and $D_{\alpha p}^0(\omega)$.

Equations (1), rewritten for all the considered values of the indices, take in the matrix notation the following form of large linear systems:

$$\begin{cases} \tilde{\mathbf{B}}^M(\omega) = \mathbf{B}^0(\omega)\mathbf{f}(\omega) \\ \tilde{\mathbf{D}}^M(\omega) = \mathbf{D}^0(\omega)\mathbf{f}(\omega) \end{cases} \qquad \begin{cases} \mathbf{a}^M(\omega) = \mathbf{B}_p^0(\omega)\,p(\omega) \\ \mathbf{u}^M(\omega) = \mathbf{D}_p^0(\omega)\,p(\omega) \end{cases} \tag{2}$$

In order to isolate the substructure, excited by $p(\omega)$, from the global structure, virtual supports are added in the form of certain virtual loads $\mathbf{f}^0(\omega)$. The responses of the structure modified this way can be expressed as the following combination of the measured original responses to $p(\omega)$ and the effect of the virtual loads:

$$\begin{cases} \mathbf{a}(\omega) = \mathbf{a}^M(\omega) + \mathbf{B}^0(\omega)\mathbf{f}^0(\omega) \\ \mathbf{u}(\omega) = \mathbf{u}^M(\omega) + \mathbf{D}^0(\omega)\mathbf{f}^0(\omega) \end{cases} \tag{3}$$

To isolate virtually the substructure, we use the boundary condition $\mathbf{a}(\omega)=0$. In this case, the responses $\mathbf{u}(\omega)$ of the isolated substructure can be constructed using the measured frequency responses to $p(\omega)$, see Eq.(4). Eq.(4) is called the isolation function.

$$\mathbf{u}(\omega) = \mathbf{u}^M(\omega) - \tilde{\mathbf{D}}^M(\omega)\left[\tilde{\mathbf{B}}^M(\omega)\right]^{-1}\mathbf{a}^M(\omega) \tag{4}$$

FFT of the Measured Response

The responses in the isolation function (Eq.(4)) are in frequency domain. They can be computed via the Fourier transform of the time-domain measurements,

$$\mathbf{x}(\omega) = \mathrm{F}\left[x(t)\right] = \int_0^{+\infty} x(t)e^{-j\omega t}dt \tag{5}$$

where F is the Fourier operator. However, when the time-domain signal is of a finite length and does not tend to zero during the integration time, the spectral leakage is avoidless. It will affect the accuracy of the constructed frequency response of the isolated substructure using the isolation function. In this case, the Laplace transform can be used instead of the Fourier operator:

$$\bar{\mathbf{x}}(s) = \mathrm{L}\left[x(t)\right] = \mathrm{F}\left[x(t)e^{-\sigma t}\right] = \int_0^{+\infty} x(t)e^{-st}dt \tag{6}$$

where L is the Laplace operator, and $s=j\omega+\sigma$. The Laplace transform is equivalent to the Fourier transform with an exponential window $w(t)=e^{-\sigma t}$.

Moreover, in real application the measured response is discrete, so the Fast Fourier Transform (FFT) is used practically to compute the frequency response.

In a word, the relative frequency responses on the right-hand side of the isolation function Eq.(4) are computed by the FFT of the finite length and discrete measured responses with an exponential window.

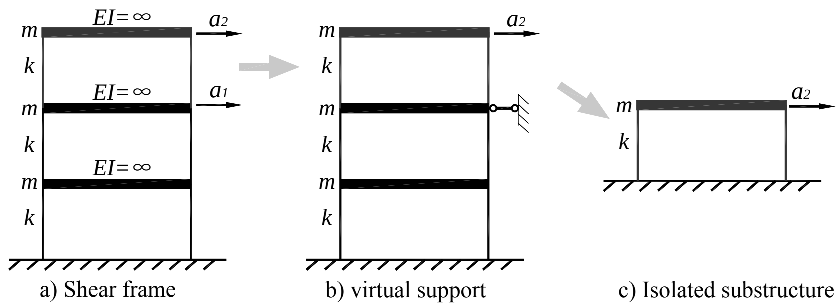

Figure 1. Shear frame structure and the isolated substructure

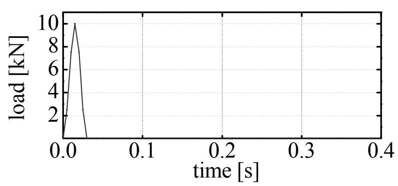

Figure 2. Excitation

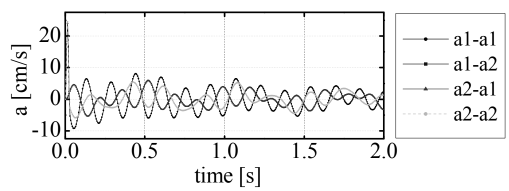

Figure 3. Responses

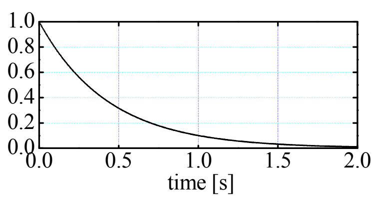

Figure 4. Exponential window

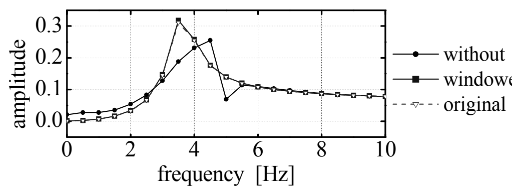

Figure 5. Frequency response

NUMERICAL EXAMPLE

A shear frame structure with three floors, shown in Figure 1(a), is taken as a numerical example to introduce the isolation substructure method using the FFT of the responses. The stiffness of each floor is k=2×10^{8}N/m, and the mass of each floor is m=4×10^{5}kg, with the first and second order damping ratio of 0.01.

The upper frame is used as the substructure to be isolated. Two accelerometers, denoted by a_1 and a_2, are placed on the second and third floor, see Figure 1(a). In order to isolate the upper frame, a virtual support is constructed at the place of accelerometer a_1 (Figure 1(b)). The isolated substructure is shown in Figure 1(c); its eigenfrequency is 3.56 Hz.

The excitation, shown in Figure 2, simulates a hammer excitation. Sampling frequency is 200 Hz, and the total time is 2 s. The excitation is applied on the positions of a_1 and a_2; the corresponding two groups of responses of the two sensors are shown in Figure 3. In order to reduce the spectral leakage, the exponential window is used, which can be seen in Figure 4. The frequency response of the isolated substructure, shown in Figure 5, is constructed by the FFT of the responses shown in Figure 3, with and without the exponential window. As we can see, the constructed frequency response without the exponential time window is incorrect.

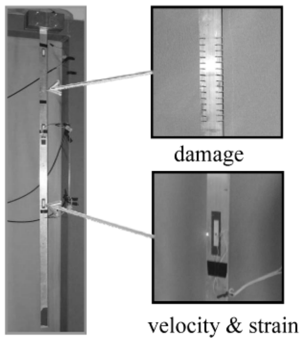

Figure 6. Cantilever beam

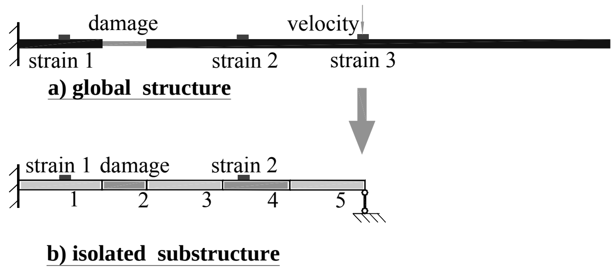

Figure 7. Isolation of the substructure

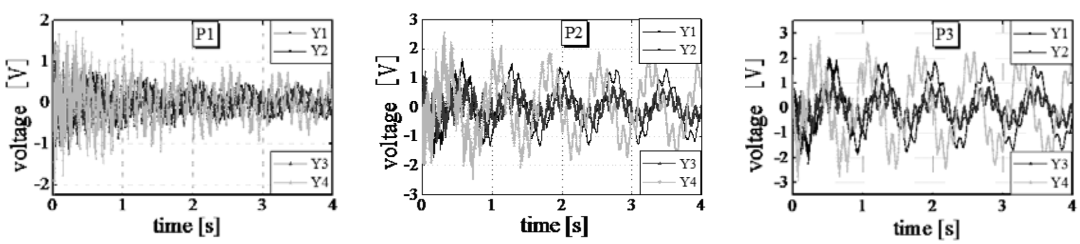

Figure 8. Measured responses

EXPERIMENT

An aluminum cantilever beam, of which the upper part is the considered substructure, is used for experimental verification, see

Figure 6. Three strain sensors (Y1, Y2, Y3) are placed on the substructure, of which one (Y3) is placed on the boundary. The boundary velocity (Y4) is measured by laser vibrometer. The single virtual pinned support which constrains the boundary responses at sensors (Y3 & Y4), see Figure 7, is used to isolate the substructure which is divided into five parts to be identified. The real damage extents of the five parts are [1 0.42 1 1 1].

The three groups of responses listed in Figure 8 were excited by hammer respectively on inner, boundary and outside of substructure, denoted by P1, P2 and P3 accordingly. The frequency responses of the isolated substructure, seen in Figure 9, were constructed using the FFT of the measured responses. The eigenfrequencies of the substructure were easily obtained via the peaks of its constructed frequency responses (Figure 9). The damages of the substructure were then identified by minimizing the square distance between the constructed eigenfrequencies of the isolated substructure and the eigenfrequencies computed using its Finite Element model, see Figure 10. Both the isolation and the identification steps are performed very well using the experimental data.

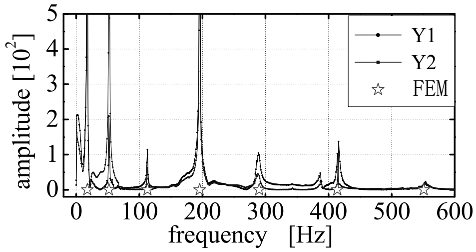

Figure 9. Constructed frequency responses

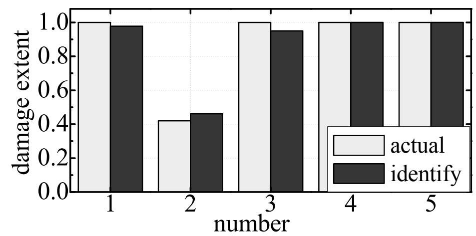

Figure 10. Damage extent

CONCLUSION

An efficient and implementation-ready method for substructure isolation using the FFT of measured responses of the global structure has been proposed and experimentally validated. The method can be applied for local structural health monitoring and damage identification.

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